§ 3.1: Polynomial Functions and Their Graphs

Polynomial Functions

A **polynomial function of degree n** is a function of the form

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where n is a non-negative integer and $a_n \neq 0$.

The numbers $a_n, a_{n-1}, \ldots, a_1, a_0$ are called the **coefficients** of the polynomial.

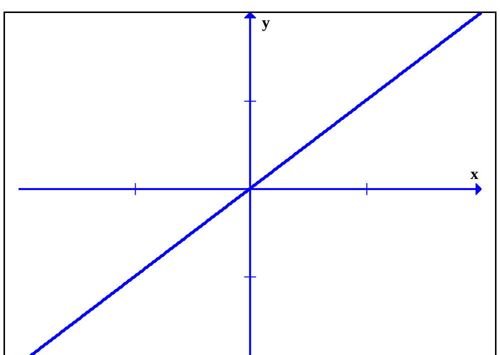
The number a_0 is called the **constant coefficient** or **constant term**.

The number a_n, the coefficient of the highest power, is the **leading coefficient**, and $a_n x^n$ is called the **leading term**.

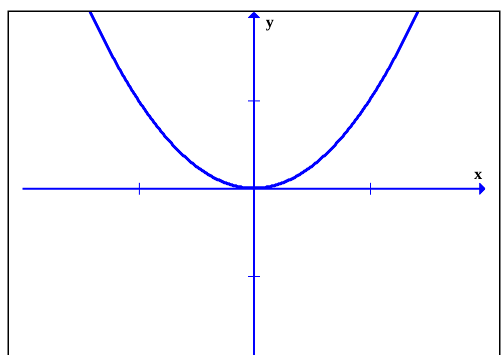
Graphs of Polynomials

The graphs of polynomials of degree 0 or 1 are lines and the graphs of polynomials of degree 2 are parabolas. As the degree of the polynomial increases, the graphs become more complex.

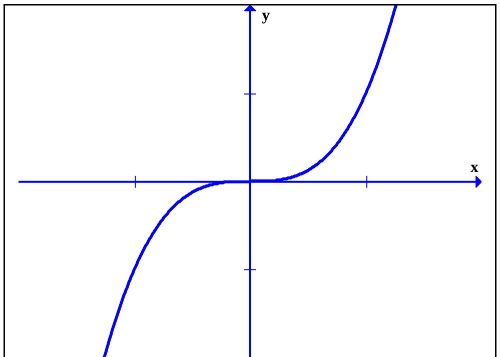
The graphs of several polynomial functions are shown below:



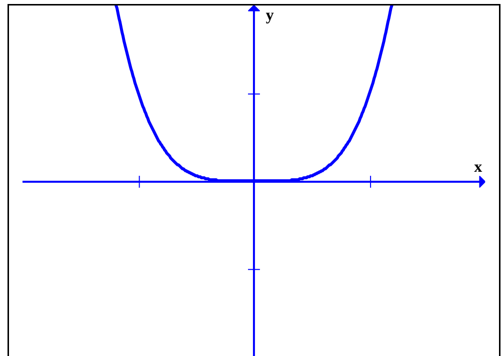
$y = x$



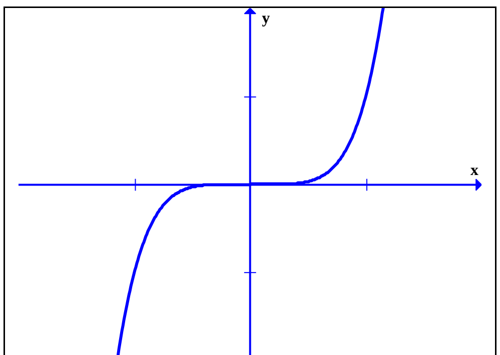
$y = x^2$



$y = x^3$



$y = x^4$



$y = x^5$

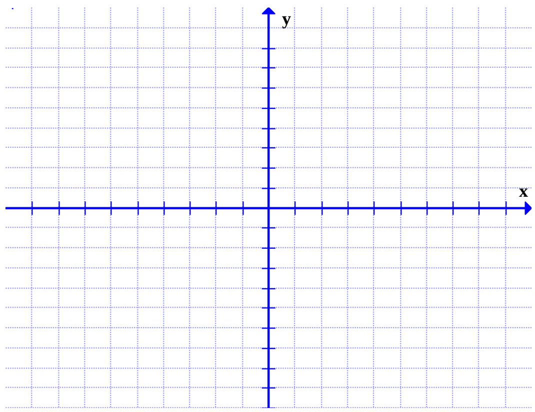
Example 1	Transformations of Monomials

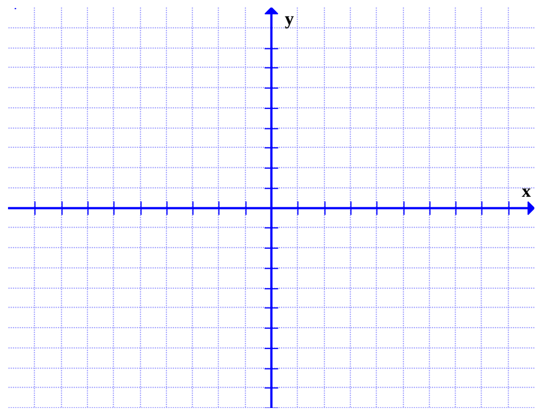
Sketch the graphs of the following functions

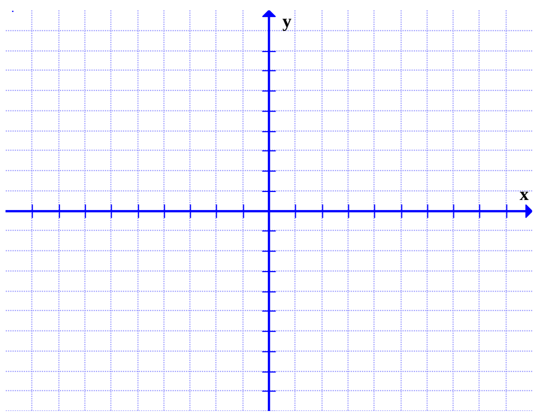
(a) $P(x) = -x^3$

(b) $P(x) = (x-2)^4$

(c) $P(x) = -x^5 + 1$







End Behavior and the Leading Term

The **end behavior** of a polynomial is a description of what happens as x becomes large in the positive or negative direction.

$x \to \infty$ means "x becomes large in the positive direction"

$x \to \infty$ means "x becomes large in the negative direction"

For example, the monomial $P(x) = x^2$ has the following end behavior

$$y \to \infty \text{ as } x \to \infty \quad \text{and} \quad y \to \infty \text{ as } x \to -\infty.$$

The monomial $P(x) = x^5$ has the following end behavior

$$y \to \infty \text{ as } x \to \infty \quad \text{and} \quad y \to -\infty \text{ as } x \to -\infty.$$

For any polynomial, the end behavior is completely determined by the leading term.

End Behavior of Polynomials

The following table completely describes the end behavior of any polynomial P:

	P has odd degree	P has even degree
Positive lead coefficient	$y \to \infty$ as $x \to \infty$ and $y \to -\infty$ as $x \to -\infty$	$y \to \infty$ as $x \to \infty$ and $y \to \infty$ as $x \to -\infty$
Negative lead coefficient	$y \to -\infty$ as $x \to \infty$ and $y \to \infty$ as $x \to -\infty$	$y \to -\infty$ as $x \to \infty$ and $y \to -\infty$ as $x \to -\infty$

Example 2	End Behavior of a Polynomial

Determine the end behavior of the polynomial

$$P(x) = 456x^{1001} + 234x^{500} + x^{400} + 3x + 5$$

Using Zeros to Graph Polynomials

If P is a polynomial function, then c is called a **zero** of P is $P(c) = 0$.

Real Zeros of Polynomials

If P is a polynomial and c is a real number, then the following are equivalent.

1. c is a zero of P.

2. $x = c$ is a solution of the equation $P(x) = 0$.

3. $x - c$ is a factor of $P(x)$.

4. $x = c$ is an x-intercept of the graph of P.

Intermediate Value Theorem for Polynomials

If P is a polynomial function and $P(a)$ and $P(b)$ have different signs, then there exists at least one value c between a and b for which $P(c) = 0$.

Guidelines for Graphing Polynomial Functions

1. Zeros Factor the polynomial to find all its real zeros; these are the x-intercepts of the graph.
2. Test Points Make a table of values for the polynomial. Include test points to determine whether the graph of the polynomial lies above of below the x-axis on the intervals determined by the zeros. Include the y-intercept in the table.
3. End Behavior Determine the end behavior of the polynomial.
4. Graph Plot the intercepts and other points you found in the table. Sketch a smooth curve that passes through these points and exhibits the requires end behavior.

Example 3	Using Zeros to Graph a Polynomial Function

Sketch the graph of the polynomial function $P(x) = (x + 1)(x - 1)(x + 2)$.

Example 4	Finding Zeros and Graphing a Polynomial

Let $P(x) = x^3 - 2x^2 - 3x$. Find the zeros of P and then sketch its graph.

Example 5	Finding Zeros and Graphing a Polynomial Function

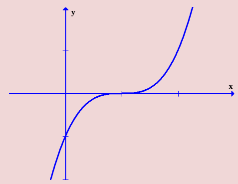
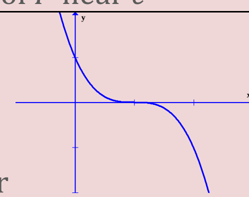
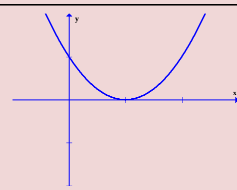
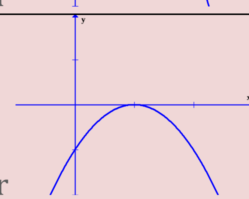
Let $P(x) = x^3 - 2x^2 + 4x + 8$. Find the zeros of P and then sketch its graph.

Shape of the Graph Near a Zero

If c is a zero of the polynomial P, then the **multiplicity** of c is the number of time the factor $x - c$ appears in the factored form of P.

Shape of the Graph Near a Zero of Multiplicity m

Suppose that c is a zero of P os multiplicity m.

Multiplicity of c	Shape of the graph of P near c
m odd, $m > 1$	 or 
m even, m > 1	 or 

Example 6	Graphing a Polynomial Function Using Its Zeros

Graph the polynomial $P(x) = x^4(x-2)^2(x+1)^2$

Homework

Due:______________________________

6 – 46 (even)